Elimination of ambiguities in $\pi\pi$ amplitudes using Roy's equations *

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Abstract

Roy's equations are used to check if scalar-isoscalar $\pi - \pi$ amplitudes fitted to the "up-flat" and "down-flat" phase shift solutions fulfill crossing symmetry below 1 GeV. It is shown that the amplitude fitted to the "down-flat" solution satisfies crossing symmetry while the "up-flat" one does not. In such a way the "up-down" ambiguity in the scalar-isoscalar phase shifts is resolved in favour of the "down-flat" solution.

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Direct study of the $\pi\pi$ scattering is beyond present experimental possibilities. However, phenomenological phase shifts can be obtained through partial wave analyses of final states of reactions in which pions are produced. These analyses are often model dependent and can lead to ambiguous results.

In 1997 a study of the $\pi^- p \uparrow \rightarrow \pi^+\pi^- n$ reaction on a polarized target was performed for the $m_{\pi\pi}$ effective mass between 600 and 1600 MeV leading to four solutions for the $\pi\pi$ scalar-isoscalar phase shifts below 1 GeV [1]. Using the unitarity constraint two "steep" solutions were rejected while the two remaining ones, called "down-flat" and "up-flat", passed this test [2].

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In order to eliminate this "up-down" ambiguity one can check if the corresponding amplitudes satisfy crossing symmetry. Roy's equations [3] can serve as a tool to perform this check and to correlate scalar-isoscalar, scalar-isotensor and vector-isovector phase shifts determined near the $\pi\pi$ threshold and at higher energy. Experimentally determined phase shifts δ and inelasticities η are used to calculate the imaginary parts of the amplitudes which can be inserted into Roy's equations. The resulting real parts of the amplitudes, called "out" can be compared with the real parts, $\eta(\sin 2\delta)/2$, directly calculated from the phase shifts and called "in". We check the quantitative agreement between the "in" and "out" real parts to test how well a given set of amplitudes satisfies Roy's equations for $m_{\pi\pi} < 970$ MeV.

A comprehensive analysis of Roy's equations with a special emphasis on the $m_{\pi\pi}$ range from threshold to 0.8 GeV has recently appeared [4]. Here we pay particular attention to the range between 0.8 and 1 GeV where the largest differences between the "up-flat" and "down-flat" solutions occur. Below 970 MeV we parameterize the amplitudes corresponding to the "down-flat" and "up-flat" data using Padé's approximants with 8 free parameters. Above 970 MeV we use the two corresponding amplitudes A and C of our analysis with three coupled channel interactions ($\pi\pi$, $K\bar{K}$ and an effective 4π system) [5]. At 970 MeV the Padé and the A or C amplitudes are smoothly connected by a proper choice of two parameters in Padé's formula. The remaining 6 parameters are obtained in fitting both experimental data and Roy's equations up to 970 MeV. Near the $\pi\pi$ threshold we use the data of [6] and above 600 MeV the "up-flat" and "down-flat" phase shifts of [1]. The S-wave isotensor amplitude is parameterized using the rank-two potential model [5] with 4 parameters fitted to the data set A of [7]. The P-wave amplitude is parameterized as in [4] and its 5 parameters are fitted to data and to satisfy Roy's equations.

For the "down-flat" case we have obtained the isoscalar amplitude describing well the experimental data ($\chi^2 = 16$ for 18 points) and simultaneously fulfilling Roy's equations. The differences between the real part values "in" and "out" were smaller than 10^{-4} for all three $\pi\pi$ amplitudes. In the "up-flat" case such a fit could not be obtained since the above differences for the isoscalar wave were as large as 0.2 around 900 MeV.

We have studied the influence of the experimental errors on the $\pi\pi$ input amplitudes by calculating Roy's equations for two extreme isoscalar amplitudes fitted to the data points shifted upwards (upper "in") or downwards (lower "in") by their errors. Below 600 MeV these fits are constrained to approximate the previously obtained “down-flat” amplitude and in particular to reproduce its scattering length $a_0^0 = 0.224$ and the slope parameter $b_0^0 = 0.272$. As seen in Fig. 1, in the “down-flat” case, both “out” curves lie inside the band limited by “in” curves up to 930 MeV. On the contrary, in the “up-flat” case above 850 MeV, the “out” band lies outside the “in” band.

We conclude that the amplitude corresponding to the “down-flat” solution does fulfill Roy’s equations but the “up-flat” amplitude does not. Thus one should accept the “down-flat” solution as the physical one and reject the “up-flat” solution. In this way the “up-down” ambiguity is resolved in favour of the “down-flat” solution as it has also been shown in a recent joint analysis of the $\pi^+\pi^-$ and $\pi^0\pi^0$ data [8].

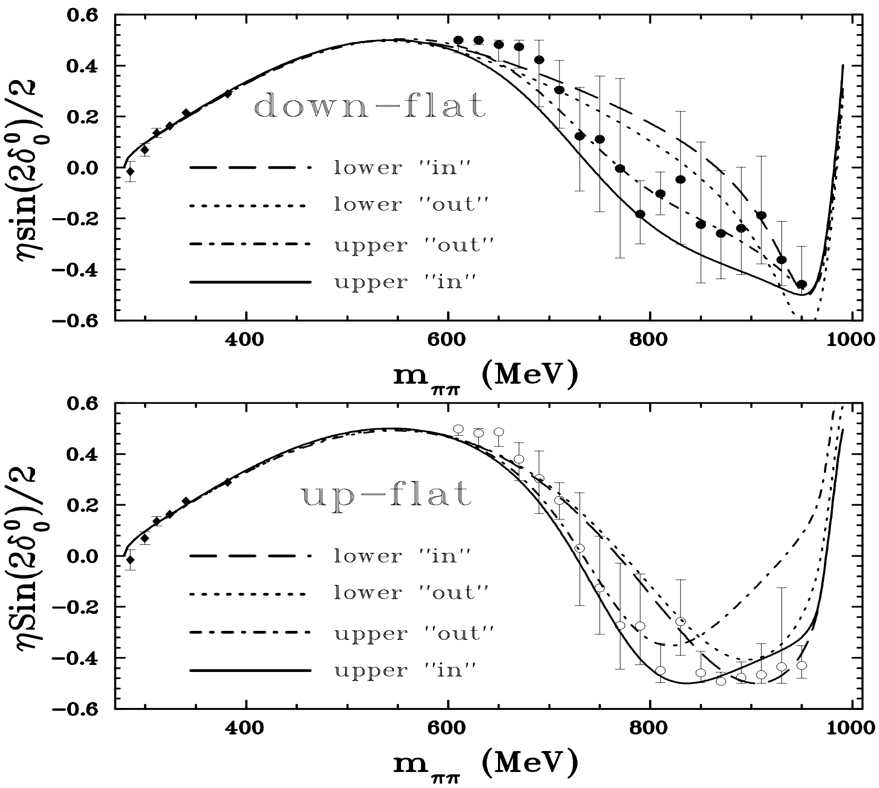

Fig. 1: Input bands (solid and dashed lines) and output bands (dotted and dot-dashed lines) computed from Roy’s equations for the S-wave isoscalar amplitude. Diamonds denote the data of [6] and circles those of [1].

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